GIMAS7AC PARTIAL DIFFERENTIAL EQUATIONS

GIMAS7AC Partial differential equations	**ECTS Credits:** 4 **Duration:** 42 hours	**Semester:** S7

Person(s) in charge:

Antoine HENROT, Professor, antoine.henrot@mines-nancy.univ-lorraine.fr

Keywords: Partial differential equations, variational formulation, boundary-value problems, spectral theory

Prerequisites: First year Math I and Math II courses of the core curriculum

Objective: To be able to analyze a well-posed problem modeled by partial differential equations

Program and contents:

Objectives

Partial differential equations are now found in models of most physical phenomena, as well as in economics, chemistry, biology… This course is designed to introduce the mathematical study of linear partial differential equations. Emphasis will be placed on questions of the existence and uniqueness of solutions as well as qualitative aspects. A project will be carried out using the Matlab «partial differential equations» toolbox. Evaluation is by continuous assessment and written tests.

Content

Introduction to elliptic, parabolic and hyperbolic partial differential equations. Examples: Laplace and Poisson equations, heat equation and wave equation. The different kinds of boundary conditions (Dirichlet, Neumann, Robin). Initial conditions for evolution problems.

Some methods for an explicit resolution: separation of variables, Fourier or Laplace transform.

Tools for the mathematical study of p.d.e.: distributions and the Sobolev spaces, Poincaré inequalities, Sobolev and compact embedding, trace.

Variational formulation of elliptic problems. The Lax-Milgram theorem. Application to different kinds of boundary conditions. Regularity results, relation between weak and classical solutions. Examples of the Laplace operator, the Stokes system, the plate equation.

Maximum principle. Spectral theory for elliptic operators, Galerkin method.

Evolution equation: the heat equation and the wave equation. Existence and regularity results, asymptotic behavior.

Assessment methods

2 written tests

Abilities:

Levels	Description and operational vocabulary
Know	To be able to recognize the different kind of partial differential equations, of boundary conditions
Understand	Understand if the problem is well posed (existence, uniqueness of a solution, stability with respect to data)
Apply	To be able to write a variational formulation in order to apply the fundamental Lax-Milgram Theorem
Analyze	To be able to study this variational formulation to prove the well-posedness of the problem
Summarize	State a correct answer to the problem, analyze a model to decide whether it is a good model
Assess	To be able to analyze the (numerical) solution of an equation to decide if it is pertinent

Evaluation:

☑ Written test	☐ Continuous assessment	☐ Oral presentation	☑ Project	☐ Written report